

BOUNDARY LAYER IN COMPRESSIBLE FLOW OF WET WATER VAPOUR

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Abstract—In this paper the influence of small droplets, with radius $10^{-8} \text{ m} < r < 10^{-6} \text{ m}$, on laminar and turbulent boundary layer behavior is considered. It is found that the laminar boundary layer in a two-phase flow with strongly dispersed liquid retains dissipation energy and that the recovery factor of enthalpy is greater than unity. In turbulent boundary layers small droplets are transported by turbulent diffusion and this leads to the recovery factor being less than unity. Its value in both cases depends mainly on the nondimensional number $Ds = C_{1,r} L / (U_i^2 / 2)$. The laminar boundary layer solution for non-equilibrium two-phase flow is obtained. Profiles of the droplet mass fraction, vapour and droplets temperatures and droplet radius are computed for the case of a steady two-dimensional flow. The turbulent boundary layer is treated using a semi-empirical theory assuming thermodynamic equilibrium.

1. INTRODUCTION

The flow of wet steam in the blade passages of a turbine is associated with deposition of droplets on the blade surface. Any adequate theory of deposition in high-speed flow must take into account the behavior of the wet vapour in the boundary layer. It is known that in high speed flow of a perfect gas dissipation causes the temperature increase at the wall. In two-phase flow evaporation of droplets in the boundary layer is possible and it alters the enthalpy distribution across layer and the temperature achieved on an insulated wall.

The problem is significant not only in deposition studies but also for heat transfer and temperature measurements.

Experimental investigations give only limited information due to the difficulty of measurement in wet steam flows and in the thin boundary layers. With regards to these difficulties simple investigations of two-phase flow in Laval nozzles have been made by Stodola (1945), Ryley (1960), Gyarmathy & Meyer (1965), and other investigators. Classical measurements of parameters such as static pressure and adiabatic wall temperature were linked with the visual observations. The existence of a superheated sublayer on a nozzle wall or probe has been confirmed by Ryley (1960), Gyarmathy & Meyer (1965), Zhukousky *et al.* (1965) and Studziński (1971).

The first theoretical estimation of boundary layers properties in high speed two-phase flow was made by Ryley (1971). However, this approach takes into account only the integral characteristics of one-phase boundary layers and does not explain the existence of a superheated layer on a nozzle wall in a wet steam flow.

The aim of the first part of this paper is to give information about the process which leads to the high values of recovery factor obtained in laminar flows. Thus the emphasis has been placed on the qualitative aspects of the solution rather than on the prediction of an actual physical flow. The physics of the process is applicable also to the laminar sublayer of a turbulent boundary layer. The second part of this paper investigates the influence of drops on the recovery factor in a turbulent boundary layer.

The steady two-dimensional two-phase flow of a compressible, viscous fluid along an insulated plate will be considered. Furthermore, a monodisperse droplet population with radius smaller than $1 \mu\text{m}$ is assumed. For these minute droplets the Magnus, gravitational and centrifugal forces are negligible and the droplets can be assumed to move without slip. The influence of Brownian diffusion for equilibrium flow has been considered by Studziński (1975) and it was found that for droplets with radius $r > 10^{-9} \text{ m}$ its significance is negligible.

2. GOVERNING EQUATIONS FOR LAMINAR BOUNDARY LAYER

The laminar boundary layer consists of a dry sublayer next to the wall and a wet steam layer. A model with no discontinuity between the wet and dry layers will be considered. The governing equations were obtained by Studziński (1975) in the classical manner from conservation equations for a multiphase mixture with $m - 1$ groups of droplets. There are three equations describing the mixture flow:

Continuity equation

$$\frac{\partial(\rho U_x)}{\partial x} + \frac{\partial(\rho U_y)}{\partial y} = 0 \quad [1]$$

where $\rho = \rho_G/[1 + (\rho_G/\rho_L - 1) \sum_{i=2}^m C_i]$ is the density of the two-phase mixture, U_x and U_y its velocity components, C_i is i th droplet group mass fraction, ρ_G and ρ_L are the specific densities of the vapour and water, respectively.

Momentum equation

$$\rho U_x \frac{\partial U_x}{\partial x} + \rho U_y \frac{\partial U_y}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial U_x}{\partial y} \right) - \frac{\partial p}{\partial x} \quad [2]$$

where p is the pressure, μ is the dynamic viscosity of the mixture which for low droplet concentration is equal to the vapour viscosity $\mu \approx \mu_G$.

Energy equation

$$\rho U_x \frac{\partial}{\partial x} \left(i + \frac{U_x^2}{2} \right) + \rho U_y \frac{\partial}{\partial y} \left(i + \frac{U_x^2}{2} \right) = \frac{\partial}{\partial y} \left(\mu \frac{\partial i_G}{\partial y} + \mu \frac{\partial U_x^2/2}{\partial y} \right) \quad [3]$$

where

$$i = \sum_{i=2}^m C_i i_i + C_G i_G$$

is the mixture enthalpy per mass unit, $i_G = c_{pG} T_G$ is the vapour enthalpy per mass unit, c_{pG} is the specific heat of the vapour at constant pressure, T_G is the vapour temperature, $i_i = c_{pL} T_{Li} - L + (c_{pG} - c_{pL}) T_s$ is the enthalpy of the i th droplet group, L is the latent heat, T_{Li} is the temperature of i th droplet group, T_s is the saturation temperature and c_{pL} is the specific temperature of water.

The mass fraction of i th droplet group depends on the concentration n_i per volume unit, on droplets radius r_i and on densities ρ and ρ_L :

$$C_i = \left(\frac{n_i}{\rho} \right) \cdot \frac{4}{3} \pi r_i^3 \cdot \rho_L. \quad [4]$$

At low pressure $\rho_G/\rho_L \ll 1$ and the mixture density has the form:

$$\rho = \frac{\rho_G}{1 - C_L}; \quad C_L + C_G = 1$$

where $C_L = \sum_{i=2}^m C_i$ is the total droplet mass fraction, and C_G is the vapour mass fraction.

For the dispersed phase (*i*th group of droplets) the governing equations are:

Concentration equation

$$\frac{\partial(n_i U_x)}{\partial x} + \frac{\partial(n_i U_y)}{\partial y} = 0. \quad [6]$$

Conservation of mass or equation of droplet growth:

$$U_x \frac{\partial r_i}{\partial x} + U_y \frac{\partial r_i}{\partial y} = \frac{2\alpha_c}{2 - \alpha_c} \frac{m}{\rho_L \sqrt{2\pi mk}} \left[\frac{p}{\sqrt{T_G}} - \frac{p_s(T_{Li})}{\sqrt{T_{Li}}} \exp\left(\frac{2\delta}{\rho_L R r_i T_{Li}}\right) \right] \quad [7]$$

where α_c is the condensation coefficient, k Boltzman's constant, m is the mass of a water molecule, R the gas constant of water vapour, $p_s(T_{Li})$ is the saturation pressure for temperature of *i*th droplet group and σ is the surface tension.

Conservation of energy or equation of droplet temperature:

$$U_x \frac{\partial T_{Li}}{\partial x} + U_y \frac{\partial T_{Li}}{\partial y} = \frac{3L}{r_i c_{pL}} \frac{2\alpha_c}{2 - \alpha_c} \frac{m}{\rho_L \sqrt{2\pi mk}} \left[\frac{p}{\sqrt{T_G}} - \frac{p_s(T_{Li})}{\sqrt{T_{Li}}} \exp\left(\frac{2\sigma}{\rho_L R r_i T_{Li}}\right) \right] - \frac{3}{r_i} \left(1 + \frac{1 - \alpha_c}{\alpha_c} \alpha_a\right) \frac{(c_{pG} - R/2)m}{\rho_L c_{pL} \sqrt{2\pi mk}} \frac{p(T_{Li} - T_G)}{\sqrt{T_G}} \quad [8]$$

where α_a is the accomodation coefficient.

The Hertz-Knudsen model of droplet growth proposed by Puzyrewski & Krol (1976) is used. The momentum equations for the drops is not used because of the zero slip assumption. The boundary conditions imposed on the set of equations [1]-[8] are:

$$\begin{aligned} y=0 \quad U_x = U_y = 0 \quad \frac{\partial i_G}{\partial y} = 0 \quad (\text{adiabatic wall}) \\ y \rightarrow \infty \quad U_x \rightarrow U_e \quad i_G \rightarrow i_{Ge} \quad r_i \rightarrow r_{ie} \\ T_{Li} \rightarrow T_{Lie} \quad n_i \rightarrow n_{ie} \end{aligned} \quad [9]$$

where quantities with subscript *e* refer to the edge of the boundary layer.

In general form there are equations for *m* groups of droplets.

A model gas is chosen with the properties:

$$p = \rho_G R T_G, \quad \mu_G = A_0 \mu_{G0} \frac{T_G}{T_{G0}}, \quad \text{Pr} = 1 \quad [10]$$

where A_0 is a constant, μ_{G0} and T_{G0} are dynamic viscosity and temperature for stagnation conditions respectively, Pr is the Prandtl number of the vapour.

3. SIMILARITY ANALYSIS OF BOUNDARY VALUE PROBLEMS

The conditions for self-similar solutions for compressible boundary layer flows impose conflicting demands on the equations for mixture and droplet phase and such solutions cannot be obtained.

In the case when the self-similarity requirements are not satisfied and parameters at the boundaries are weakly changable, the concept of local self-similarity has been applied quite successfully to solve the problem. These solutions show most of the characteristics of the flow and the influence of parameter variation on the boundary layer flow behaviour without introducing the complications of non-similar solutions.

After analysis of the governing equations with one group of drops the new independent transformed variables are:

$$\zeta = \frac{\rho_e U_e}{\sqrt{(2A_0 \rho_e U_e x_0 \mu_{G0} \bar{\xi})}} \int_0^y \frac{\rho_G}{\rho_{Ge}} dy \quad [11]$$

$$\bar{\xi} = x/x_0$$

where x_0 is the characteristic length of the plate.

The new independent variable is similar to Lees (1956) variable, however the integrated function is not the mixture but the gas phase density. It was impossible to obtain a solution with the classical transformation where ρ is the density of mixture.

The introduction of a stream function ψ

$$\psi(x, y) = \sqrt{(2\mu_{G0} A_0 U_e \rho_{Ge} x)} K(\zeta) \quad [12]$$

together with the definitions

$$U_x(x, y) = U_e \bar{U}(\zeta); \quad r(x, y) = r_e \bar{r}(\zeta); \quad T_G(x, y) = T_{Ge} \bar{T}_G(\zeta);$$

$$T_L(x, y) = T_{Le} \bar{T}_L(\zeta); \quad \left(\frac{n_L}{\rho}\right) = \left(\frac{n_L}{\rho}\right)_e \bar{N}(\zeta) \quad [13]$$

where on the edge of boundary layer U_e , r_e , T_{Ge} , T_{Le} , $(n/\rho)_e$ are constant, leads to the conservation equations for locally self-similar flow in the form:

$$K''' + K''[2(\ln \bar{C}_G)' + K] + K' \left[\frac{\bar{C}_G''}{\bar{C}_G} + K(\ln \bar{C}_G)' \right] = 0$$

$$\bar{T}_G'' + K \bar{T}_G' - \text{Ec} [3K \bar{N} \bar{r}'^2 \bar{r}' \text{Ds} - 2(K'' \bar{C}_G + K' \bar{C}_G')^2] = 0$$

$$\bar{r}' = -\frac{2\bar{\xi}}{\bar{C}_G K} \text{Sd} \left(\frac{1}{\sqrt{\bar{T}_G}} - \frac{\bar{p}r}{\sqrt{\bar{T}_L}} \right) \quad [14]$$

$$\bar{T}_L' = 3 \text{Jc} \frac{\bar{r}'}{\bar{r}} - 2 \frac{\bar{\xi}}{\bar{C}_G K \bar{r}} \frac{c_{pG} - R/2}{c_{pL}} \frac{3(1 + \frac{1-\alpha_c}{\alpha_c} \alpha_a)}{2\alpha_c(2-\alpha_c)} \text{Sd} \frac{\bar{T}_L - \bar{T}_G}{\sqrt{\bar{T}_G}}$$

$$\bar{N}' = 0.$$

The boundary conditions now become:

$$\zeta = 0; \quad K = \bar{U} = 0; \quad \bar{T}_G' = 0;$$

$$\zeta = \infty; \quad \bar{U} \rightarrow 1; \quad \bar{T}_G \rightarrow 1; \quad \bar{T}_L \rightarrow 1;$$

$$\bar{N} \rightarrow 1; \quad \bar{r} \rightarrow 1. \quad [15]$$

Primes are used to denote differentiation with respect to ζ and other expressions are given as:

$$(1 - C_{Ge}) \bar{C}_G + C_{Le} \bar{C}_L = 1$$

$$\bar{C}_L = \frac{C_L}{C_{Le}} = \bar{N} \bar{r}^3$$

$$\text{Sd} = \frac{2\alpha_c}{2 - \alpha_c} \frac{\rho_e \cdot m \cdot x_0 / U_e}{\rho_L r_G \sqrt{(2\pi m k T_{Ge})}}$$

$$\text{Ds} = \frac{C_{Le} \cdot L}{U_e^2 / 2} = 1 + \frac{1}{\text{Ec}} - \frac{H_0}{U_e^2 / 2}$$

$$\text{Ec} = \frac{U_e^2 / 2}{c_p T_{Ge}}$$

$$\text{Jc} = \frac{L}{c_p T_{Ge}}$$

$$\bar{p}_r = p_s(T_L) / p_e \exp\left(\frac{2\sigma}{\rho_L R T_{Lr}}\right) \quad [16]$$

where S_d is the Stodola number, D_s is a new nondimensional number, E_c is the Eckert number and J_c is the Jacob number. These are nondimensional numbers which characterise two-phase flow. The Stodola number, S_d , expresses the ratio of droplet residence time to the evaporation time in the boundary layer.

The number D_s is the ratio of energy consumption on evaporation to kinetic energy.

4. LAMINAR BOUNDARY LAYER IN THERMODYNAMIC EQUILIBRIUM

It is possible to derive a simple integral of the energy equation for equilibrium flow. The gas phase temperature is equal to the droplet's temperature in the equilibrium state, $T_G = T_L$, and for one droplet fraction [7] has the simple form

$$U_x \frac{\partial r}{\partial x} + U_y \frac{\partial r}{\partial y} = 0. \quad [17]$$

The energy equation [3] can be rewritten as

$$\begin{aligned} \rho U_x \frac{\partial}{\partial x} \left(i_G + \frac{U_x^2}{2} \right) + \rho U_y \frac{\partial}{\partial y} \left(i_G + \frac{U_x^2}{2} \right) &= \frac{\partial}{\partial y} \left[\mu \frac{\partial}{\partial y} \left(i_G + \frac{U_x^2}{2} \right) \right] \\ &+ L \left(\rho U_x \frac{\partial C_L}{\partial x} + \rho U_y \frac{\partial C_L}{\partial y} \right). \end{aligned} \quad [18]$$

Noting [1], [6] and [7] the last term in [18] is zero.

$$\begin{aligned} \rho U_x \frac{\partial C_L}{\partial x} + \rho U_y \frac{\partial C_L}{\partial y} &= \rho U_x \frac{\partial}{\partial x} \left(\frac{n}{\rho} \right) + \rho U_y \frac{\partial}{\partial y} \left(\frac{n}{\rho} \right) + \frac{4}{3} \pi r^2 \rho_L \left(U_x \frac{\partial r}{\partial x} + U_y \frac{\partial r}{\partial y} \right) \\ &= \rho U_x \frac{\partial}{\partial x} \left(\frac{n}{\rho} \right) + \rho U_y \frac{\partial}{\partial y} \left(\frac{n}{\rho} \right) + \frac{n}{\rho} \left[\frac{\partial(\rho U_x)}{\partial x} + \frac{\partial(\rho U_y)}{\partial y} \right] \\ &= \frac{\partial(n U_x)}{\partial x} + \frac{\partial(n U_y)}{\partial y} = 0. \end{aligned}$$

For flow with zero pressure gradient [19] is similar to momentum [2]

$$\rho U_x \frac{\partial H_G}{\partial x} + \rho U_y \frac{\partial H_G}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial H_G}{\partial y} \right) \quad [19]$$

where $H_G = i_G + (U_x^2/2)$ is the gas phase total enthalpy.

In that case it has the following solution

$$H_G(x, y) = \text{const.} \quad [20]$$

From the boundary conditions

$$\begin{aligned} y = 0; \quad & \frac{\partial H_G}{\partial y} = 0 \\ y \rightarrow \infty; \quad & H_G \rightarrow H_{Ge} \end{aligned}$$

it follows that the constant will be equal to the total enthalpy of gas phase in the external flow

$$H_G = H_{Ge}. \quad [21]$$

This solution is similar to the Crocco integral for dry, perfect gas flow in a laminar boundary layer. It must be emphasized that it holds only for $(\partial p / \partial x) = 0$ and $Pr = 1$.

The temperature profile in a two phase boundary layer on an insulated wall ($Pr = 1$) is coupled with the velocity profile $U_x(x, y)$ and the wetness fraction in the external flow C_{Le} :

$$T_G = \left(H_0 - \frac{U_x^2}{2} + C_{Le} \cdot L \right) / c_{pG}. \quad [22]$$

Temperature of the insulated wall T_w is

$$T_w = (H_0 + C_{Le} \cdot L) / c_{pG}. \quad [23]$$

If stagnation parameters lie in the superheated steam field then the wall temperature is greater than stagnation temperature T_0

$$T_w = T_0 + C_{Le} \cdot L / c_{pG}. \quad [24]$$

The recovery factor r_E in general form is defined as

$$r_E = \frac{i_w - i_e}{i_0 - i_e} \quad [25]$$

where i_w is the enthalpy on the insulated wall, i_0 is the stagnation enthalpy and i_e is the mixture enthalpy in external flow.

For two-phase flow it has the form:

$$r_E = 1 + \frac{C_{Le} \cdot L}{U_e^2 / 2} = 1 + Ds. \quad [26]$$

It will be shown that the recovery factor for equilibrium flow is greater than unity and depends on the nondimensional number Ds . It is well known that for dry (superheated or supersaturated) flow and $Pr = 1$, the recovery factor is unity. Assuming $C_{Le} = 0$ we have $r_E = 1$. The radius of equilibrium droplets in the laminar boundary layer, from [7], is

$$r = \frac{2\sigma}{\rho_L L [1 - T_G / T_s(p_e)]}. \quad [27]$$

The gas phase temperature T_G increases towards the wall and the equilibrium droplet radius must increase also. It is seen from definition [4] that wetness, C_L , tends to zero when the radius or concentration tends to zero. We conclude that in thermodynamic equilibrium the droplet concentration decreases in the boundary layer causing a decrease of wetness fraction.

There are two physically possible situations. Droplets with radius r_0 are "thrown into" the laminar boundary layer from the outer edge, further they move without slip, and are evaporated (see the next chapter) because their initial radius r_0 is too small comparing with equilibrium radius r , [27].

In the second case droplets are nucleated within the boundary layer. The concentration decreases quickly towards the wall because a nucleation rate strongly depends on supersaturation $p_d/p_s(T_G)$. Behind nucleation zero there is a thin wet layer of the equilibrium droplets [27].

It is possible to estimate the location of the interface between the wet and superheated layers.

From equilibrium condition we have

$$T_L = T_G = T_s(p_e) \left(1 - \frac{2\sigma}{rL\rho_L} \right). \quad [28]$$

The temperature distribution across the boundary layer is given by [21] and for limit value $r \rightarrow \infty$ we obtain the following Crocco coordinate for interface

$$\bar{U}_{int} = \sqrt{\left(1 - \frac{c_{pG}(T_s(p_e) - T_{Ge})}{U_e^2/2}\right)} \tag{29}$$

For example for $p_e = 0.7$ bar, $r_e = 3.5 \cdot 10^{-8}$ m, $U_e = 700$ m/s we get $\bar{U}_{int} = 0.9983$. It is seen that the depth of droplet penetration in an equilibrium laminar boundary layer is negligible. There exists only a dry boundary layer and the solution of the momentum equation [2] does not differ from that for dry gas.

5. SOLUTION PROCEDURE AND RESULTS

The equations ([14]) developed in section 2 could be used to calculate the two point boundary value problem for steady, compressible, non-equilibrium two-phase flow in laminar boundary layer.

The droplet growth equations are of the first order and for liquid phase there is the initial value problem. Strong nonlinearity of the equations for droplet growth and mixed initial-boundary value problem causes, that more sophisticated method like quasi-linearization or iteration scheme applying the principle of functional iteration, is useless.

Employing the technique of repeated "shooting" from the inner boundary, it is necessary to predict initial values for radius and droplet temperature at the interface between the superheated sublayer and the wet steam layer. These values are not independent and it is impossible to foresee correctly the droplet temperature. The solution of the governing equation is obtained by modifying the droplet temperature equation. It is known that the components on the right-hand-side have absolute values much greater than the derivative of T_L . Hill (1966), in his work neglected this derivative and replaced the differential equations by algebraic equations. The assumption that the capillary effects are negligible allowed him to de-couple the equations of growth and of the droplet temperature but the accuracy of the solution was rather unsatisfactory (figures 1 and 2).

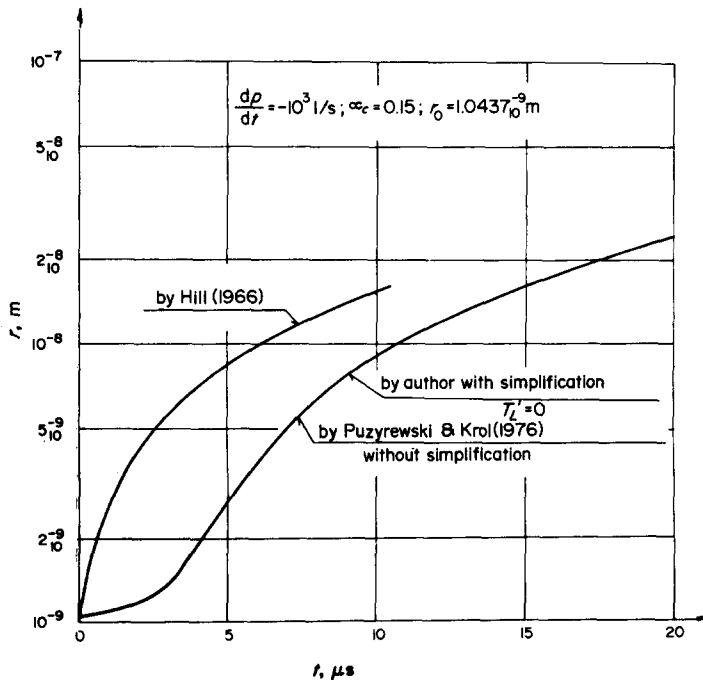


Figure 1. Comparison of Hill's and Puzyrewski & Studziński methods of droplet growth computations. Computations are for constant expansion rate dp/dt .

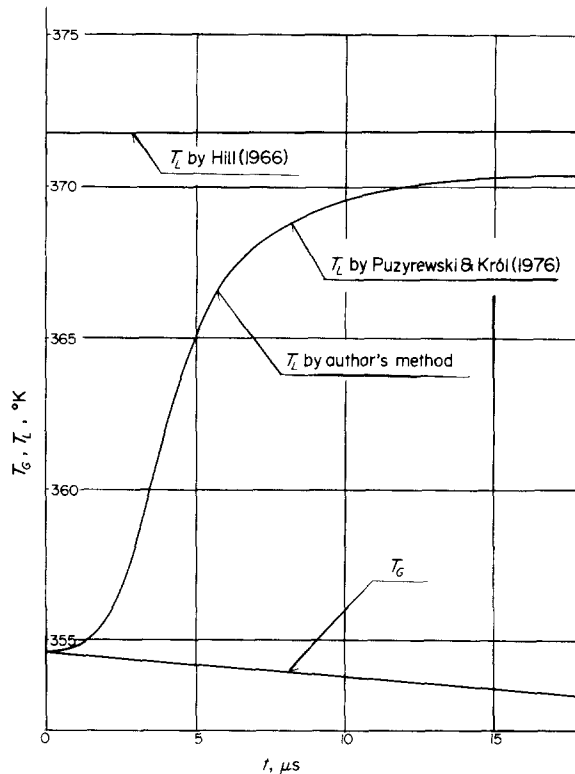


Figure 2. Comparison of Hill's and Puzyrewski's and the author's methods of droplet temperature computations.

It is proposed here to take into consideration the surface tension influence on pressure and solve simultaneously the algebraic and differential equations.

When the gas parameters and droplet radius is given, algebraic equation exactly defines the droplet temperature. In this case initial values are not required. The accuracy of the proposed method does not differ more than 0.5% for radius and 0.01% for temperature, from the unsimplified solution derived by Puzyrewski & Krol (1976).

For given boundary values on the wall, defined position of interface and small initial droplet radius the derivative $K''(0)$ and concentration (n/ρ) on interface are obtained by means of iteration.

The numerical calculations have been carried out with the Prandtl number of the gas phase equal unity for water vapour, condensation coefficient $\alpha_c = 0.15$, accommodation coefficient $\alpha_a = 1.0$.

Figure 3 shows nondimensional profiles of velocity \bar{U}_x , gas phase temperature \bar{T}_G and wetness fraction \bar{C}_L as functions of Ds number. Its increase causes greater depth of droplet penetration in non-equilibrium laminar boundary layers and increases the insulated wall temperature. Droplet evaporation does not change the velocity profiles significantly because of the rather small change of viscosity. Its influence on the skin friction coefficient C_f is negligible. There is also shown the temperature distribution \bar{T}_G for equilibrium flow with number $Ds = 0.19$. The insulated wall temperature \bar{T}_w in nonequilibrium flow with $Ds = 0.19$ is lower than for equilibrium and higher than for supercooled flow.

Dissipation energy is utilised by droplet evaporation and gas temperature gradient reductions. The heat conducted to the outer flow is reduced and this causes an increase of the wall temperature.

Comparing graphs of wetness \bar{C}_L (figure 4) or droplet radius (figure 5) against ζ for different Stodola numbers it is seen that the wetness gradient depends on the droplet dispersion. For the same external wetness but different concentrations the more dispersed mixture retains more dis-

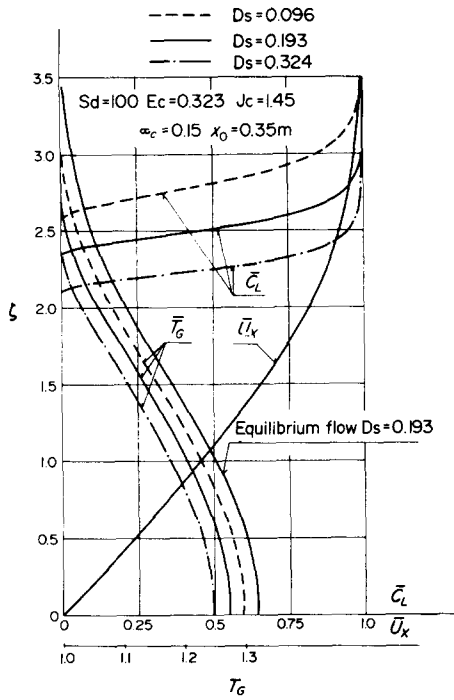


Figure 3.

Figure 3. Wetness fraction, gas temperature and velocity distribution across non-equilibrium laminar boundary layer.

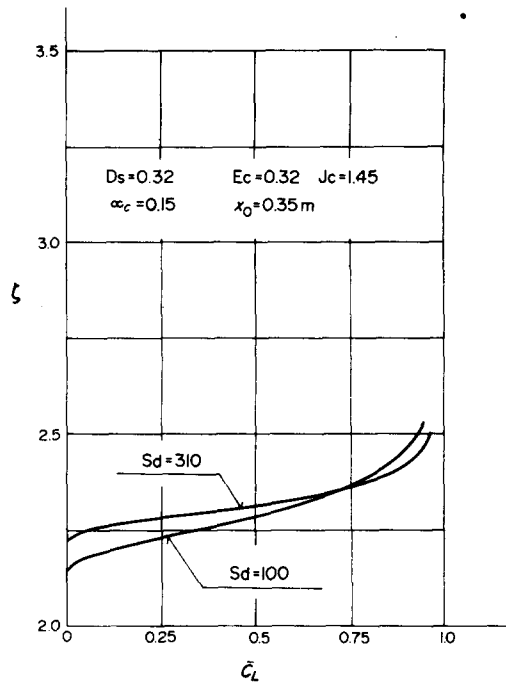


Figure 4.

Figure 4. Influence of liquid phase dispersion (Stodola number) on the depth of droplet penetration in non-equilibrium laminar boundary layer.

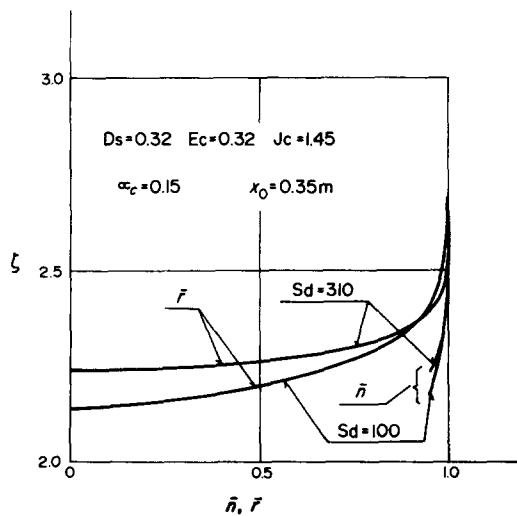


Figure 5. Radius of droplets and their concentration distribution across non-equilibrium laminar boundary layer.

sipation energy and evaporates more rapidly. The wet layer thickness is a function of D_s number, Stodola and Eckert numbers.

In figure 6 profiles of droplet temperature T_L and gas phase temperature T_G are shown. Rise of temperature T_L appears when the flux of energy, which is transferred with evaporated molecules, is weaker in comparison with the conduction flux. In the outer part of the boundary layer the temperature T_L follows T_G but droplet radius slowly diminishes. When the maximum

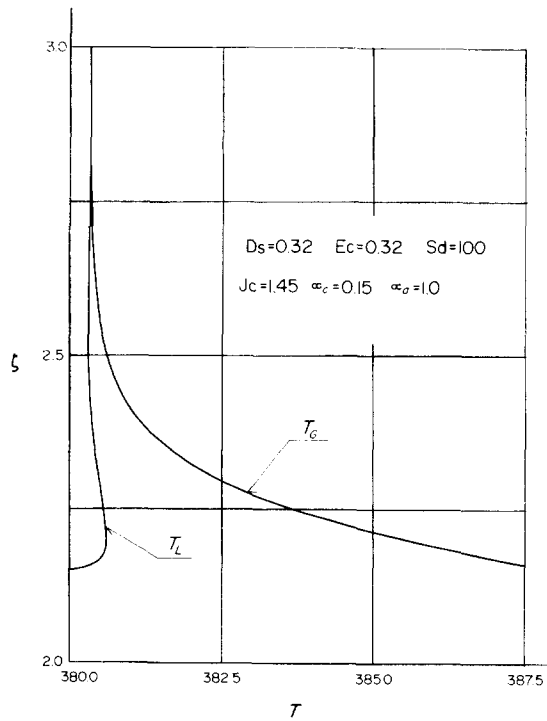


Figure 6. Gas phase and droplet temperature in non-equilibrium laminar boundary layer.

of droplet temperature T_L is achieved there is rapid decrease of T_L connected with rapid droplet evaporation.

It is convenient to introduce recovery factor of enthalpy defined by [25]. For supercooled flow of water vapour ($Pr = 1$) the recovery factor is equal to unity. Its value in equilibrium flow depends only on the D_s number [26] and is greater than unity. This effect is similar to the flow of dry gas with Prandtl number greater than unity.

Graphs of recovery factor in a laminar boundary layer for equilibrium and nonequilibrium flows are shown in figure 7 plotted against the nondimensional number D_s . Nonequilibrium evaporation has little effect on recovery factor and its value depends only weakly on Stodola, Jacob and Eckert numbers.

6. EVAPORATION OF DROPLETS IN COMPRESSIBLE TURBULENT BOUNDARY LAYER—EFFECT ON RECOVERY FACTOR

Laminar boundary layer theory in compressible two-phase flow is of rather qualitative significance. In many practical applications the boundary layer will be turbulent.

The influence of dissipation energy on the evaporation of small drops in a turbulent boundary layer is of particular interest in wet steam turbines.

The steady two dimensional flow of a compressible viscous mixture along an insulated, flat plate will be considered. Small drops, $10^{-8} \text{ m} < r < 10^{-6} \text{ m}$, in thermodynamic equilibrium and with wetness fraction $C_{L,e} \ll 1$ is assumed. The dispersed phase moves without slip. The assumption of equality of the turbulent diffusion coefficients for gas and small droplets is permissible as stated by Rouhianen & Stachiewicz (1970). Equality of gas and droplet temperatures allows the use of the theory of turbulent boundary layers with chemical reactions.

The momentum and energy equations in Crocco coordinates (x, U_x) have the general form:

$$U_x \frac{\partial}{\partial x} \left(\frac{\mu + \mu_T}{\tau} \rho \right) + \frac{\partial^2 \tau}{\partial U_x^2} = 0 \quad [30]$$

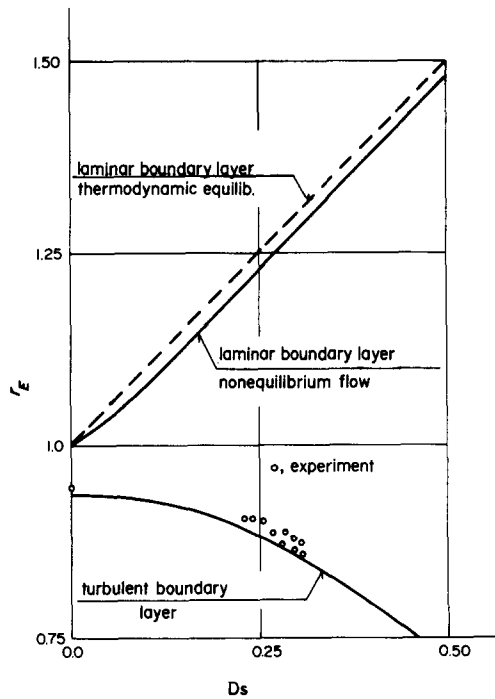


Figure 7. Recovery factor of enthalpy for laminar and turbulent boundary layers in two phase flow.

$$\rho U_x \frac{\mu + \mu_T}{\tau} \frac{\partial H}{\partial x} + \frac{\partial \tau}{\partial U_x} \frac{\partial H}{\partial x} = \frac{\partial}{\partial U_x} \left\{ \tau \left[\frac{1}{Pr_m} \frac{\partial H}{\partial U_x} + \left(\frac{1}{Sc_m} - \frac{1}{Pr_m} \right) L \frac{\partial C_L}{\partial x} + \left(1 - \frac{1}{Pr_m} \right) U_x \right] \right\} \quad [31]$$

where $H = c_p T - C_L L + U_x^2/2$ is total mixture enthalpy, $\tau = (\mu + \mu_T) \partial U_x / \partial y$ is the local shear stress, μ_T turbulent viscosity, $Pr_m = Pr$ or Pr_T and $Sc_m = Sc$ or Sc_T are the molecular or turbulent Prandtl and Schmidt numbers respectively.

For Prandtl's scheme of turbulent boundary layer structure it is necessary in the laminar sublayer to take molecular values $Pr_m = Pr$ or $Sc_m = Sc$ (Brownian diffusion) and in the fully turbulent domain $Pr_m = Pr_T$ or $Sc_m = Sc_T$.

Karman's approach allows for the third domain—the so-called buffer zone—where Pr_m , Sc_m have values changing between turbulent and laminar values.

Dissipation energy causes droplet evaporation in the turbulent boundary layer. There will exist an interface between the superheated sublayer and the wet part of the boundary layer. Because of the low droplet concentration C_{Lw} , the interface position can only be in the fully turbulent domain or in the buffer zone and the droplet influence on the velocity profile is negligible.

The main difficulty is the estimation of the turbulent Prandtl and Schmidt number distributions across the boundary layer and the proper choice of their value in each zone.

In the simplest case it is possible to assume, for the Prandtl scheme, constant values of nondimensional numbers $Pr = Pr_T = Sc_T = 1$. The interface between the wet layer and the superheated sublayer lies in the fully turbulent domain. Solution of [30] and [31], is obvious:

$$H = aU_x + b \quad [32]$$

where a , b are constants.

Using boundary conditions for an insulated wall the integral of [32] has the form:

$$H = H_e \quad [33]$$

where

$$H_e = c_{pG} \cdot T_e - C_{Le} L + \frac{U_e^2}{2} = H_0. \quad [34]$$

The temperature of an insulated wall, for an external flow which stagnation parameters p_0 , T_0 lie in the superheated field is:

$$T_w = T_0 - C_{Lo} L / c_{pG} \quad [35]$$

and the recovery factor

$$r_E = \frac{i_w - i_e}{i_0 - i_e} = 1. \quad [36]$$

The droplet concentration distribution in the wet zone of the boundary layer has the form:

$$\bar{C}_L = 1 - (1 - \bar{U}_x^2) / Ds. \quad [37]$$

Interface coordinate $(\bar{U}_x)_{int}$, in which lies the fully turbulent domain, is

$$(\bar{U}_x)_{int} = \sqrt{(1 - Ds)} \quad [38]$$

and the temperature distribution in the superheated sublayer has the form:

$$\bar{T} = 1 + (\bar{U}_{x_{int}}^2 - \bar{U}_x^2) \cdot Ec = 1 + (1 - Ds - \bar{U}_x^2) Ec. \quad [39]$$

These solutions are quite different in form from those for a laminar boundary layer obtained previously because of the turbulent diffusion of the droplets. Coefficients of momentum, energy and mass diffusion are equal ($Pr = Pr_T = Sc_T = 1$). Thus the recovery factor is equal to unity.

However, in real flows the nondimensional numbers are changing across the boundary layer and their average value in the turbulent zone is less than unity.

Recovery factor computation from [30] and [31] requires information about the turbulent viscosity μ_T in two-phase flow and the turbulent Prandtl and Schmidt numbers.

Van Driest (1959) solved a similar problem in a perfect gas flow with $Pr_m \neq 1$ in a simpler way. He assumed a logarithmic velocity profile and used shear stress distributions obtained from measurements. The influence of compressibility was taken into account only by change in local skin friction coefficient. Using the local similarity concept he assumed that, in Crocco coordinates, the enthalpy distribution depended only on U_x . It was useful simplification for the case of zero pressure gradient and constant wall temperature. The problem was reduced to the solution of an ordinary linear, differential energy equation.

The case of two-phase flow along flat insulated plate with the assumptions related previously will be treated using van Driest's approach.

The influence of droplet concentration on the shear stress distribution is taken into account. Therefore the local skin friction coefficient is modified.

For the simple Falkner law, which applies in incompressible flow

$$C_{f0} = \frac{2\tau_w}{\rho U_e^2} = 0.0263 Re_x^{-1/7} \quad [40]$$

where $Re_x = \rho U_e x / \mu$ is the Reynolds number, and the reference temperature, T_m , is defined as

$$T_a = (T_e + T_w)/2. \quad [41]$$

From the recovery factor definition [25], the ratio T_a/T_e is derived:

$$\frac{T_a}{T_e} = 1 + (r_E - Ds) \cdot Ec/2. \quad [42]$$

The assumption of a linear dependence between viscosity and temperature

$$\frac{\mu_a}{\mu_e} = \frac{T_a}{T_e} \quad [43]$$

and the perfect gas assumption

$$\frac{\rho_e}{\rho_a} = \frac{1}{1 - C_{Le}} \frac{T_a}{T_e}$$

allows the derivation of the skin friction coefficient, C_f , in compressible, wet steam flow:

$$\begin{aligned} C_f &= \frac{2\tau_w}{\rho_e U_e^2} = 0.0263 \left(\frac{\rho_e U_e x}{\mu_e} \right)^{-1/7} \left(\frac{\rho_e}{\rho_a} \right)^{-6/7} \left(\frac{\mu_a}{\mu_e} \right)^{1/7} \\ &= 0.0263 \text{Re}_x^{-1/7} \cdot (1 - C_{Le})^{6/7} [1 + (r_E - Ds)Ec/2]^{-5/7}. \end{aligned} \quad [44]$$

Van Driest's (1959) expression for shear stress distribution in a turbulent boundary layer with a logarithmic velocity profile has the form:

$$\bar{\tau} = 1 - \exp \left[-\frac{0.4}{\sqrt{(C_{f0}/2)}} (1 - \bar{U}_x) \right]. \quad [45]$$

This distribution is for incompressible flow and for compressible wet-steam flow the skin friction coefficient should be changed to $C_f \cdot (\rho_e/\rho_a)$ from expression [44].

Coordinates of the boundary between the laminar sublayer and the buffer zone, $\bar{U}_{xl} = 5\sqrt{(C_{f0}/2)}$, or between the buffer zone and the fully turbulent domain, $\bar{U}_{xt} = 5(1 + \ln 6)\sqrt{(C_{f0}/2)}$, will be modified in the same way. Using the local similarity concept the energy equation in every zone has the general form:

$$\bar{H}'' - (W_m - 1)\bar{H}'\bar{\tau}'/\bar{\tau} + (W_m - 1)(1 + \bar{U}_x\bar{\tau}'/\bar{\tau})U_e^2/H_e + W_m'/W_m(\bar{U}_x U_e^2/H_e - \bar{H}') = 0 \quad [46]$$

where W_m could be Pr , Pr_T , Sc , Sc_T respectively. There is superheated steam below the interface and in this layer must be taken $W_m = \text{Pr}$ (molecular or turbulent respectively to the zone). The wet-steam layer is in the thermodynamic equilibrium, temperature is constant $T_G = T_s(p_e)$ and conduction flux is zero. Energy transport is caused by droplet diffusion, therefore $W_m = \text{Sc}$ (molecular or turbulent). In the buffer zone nondimensional numbers have medial values $\text{Pr} \leq \text{Pr}_b \leq \text{Pr}_T$, $\text{Sc} \leq \text{Sc}_b \leq \text{Sc}_T$. Assuming constant shear stress, $\tau = \tau_w$, in that zone

$$\text{Pr}_b = \frac{\text{Pr}_T}{1 + (\text{Pr}_T/\text{Pr} - 1) \exp \left(1 - \frac{\bar{U}_x}{5\sqrt{(C_f/2)}} \right)}. \quad [47]$$

An analogous expression is derived for the Schmidt number.

There are some difficulties concerning the proper choice of turbulent Schmidt and Prandtl numbers. The distribution of turbulent Prandtl number across a boundary layer was investigated by Simpson *et al.* (1970) who derived the formula for the outer region

$$\text{Pr}_T = 0.95 - 0.45(y/\delta)^2. \quad [48]$$

where δ is the thickness of the turbulent boundary layer. The turbulent Prandtl number in expression [47] is the value on the boundary of the buffer zone and the value of $\text{Pr}_T = 0.9$ is taken from Simpson *et al.* (1970) data. In the fully turbulent domain (superheated layer) [48] is applied.

The molecular Prandtl number for water vapour is unity. The turbulent Schmidt number distribution was investigated by Dunbar & Squire (1971). The medial value in the fully turbulent zone is approximately $\text{Sc}_T \cong 0.5$ but in the buffer zone it increases, e.g. for droplets with radius $r \geq 10^{-8}$ m, to value $\text{Sc} \geq 1000$.

Results of recovery factor, r_E , computations are shown in figure 7. Recovery factor depends mainly on the nondimensional number $Ds = C_{Le} \cdot L/(U_e^2/2)$. Dependence on Eckert's and Reynold's number is negligible. Increase of the parameter Ds causes decrease of the recovery factor. This dependence occurs when the influence of the molecular Schmidt number in the buffer zone is weak.

Experimental data are taken from Studziński (1971), (1975) measurements on an insulated plate aligned with the axis of a Laval nozzle.

The results of adiabatic wall temperature measurements are similar to obtained by Ryley (1960) and Zhukousky *et al.* (1965).

The liquid phase was nucleated during homogenous condensation. Data are measured in the zone downstream of the onset of condensation, near thermodynamic equilibrium. Agreement with semi-empirical predictions has been found to be satisfactory.

7. CONCLUSIONS

The results of the analyses of this paper can be summarized as follows:

(1) The recovery factor of enthalpy in compressible wet vapour flow, with negligible Brownian diffusion, is greater than unity for laminar boundary layers and less than unity in turbulent boundary layers.

(2) The recovery factor in both laminar and turbulent boundary layers depend mainly on the nondimensional number $Ds = C_{Le} \cdot L/(U_e^2/2)$.

(3) Dispersed droplets in compressible laminar boundary layers retain dissipation energy so that the insulated wall temperature can be higher than the stagnation temperature of the external flow. The depth of droplet penetration depends on Eckert, Ds and Stodola numbers. The latter expresses the influence of droplet dispersion.

(4) Turbulent diffusion of small droplets is responsible for the thermal behaviour of the turbulent boundary layer. The droplets are strongly evaporated in the buffer zone and laminar sublayer.

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